

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Exam 1

Abstract Algebra 1

11-11-2008

Choose any 4 problems from the following 8 problems.

1. Let  $G$  be a group with identity  $e$ . Prove that if  $a^2 = e$  for all  $a \in G$  then  $G$  is abelian.
2. Let  $G$  be the set of all real numbers except  $-1$ . Define a binary operation  $\star$  on  $G$  such that  $a \star b = a + b + ab$ . Prove that  $G$  is a group.
3. Let  $G$  be a finite group and  $a \in G$ . Prove that  $\{ag \mid g \in G\} = G$ .
4. Let  $G$  be an abelian group with identity  $e$ . Prove that the set  $\{x \in G \mid x^2 = e\}$  is a subgroup of  $G$ .
5. Let  $G$  be a group and  $H \subseteq G$ . Assume that  $H$  is a finite set and  $ab \in H$  for all  $a, b \in H$ . Prove that  $H$  is a subgroup of  $G$ .
6. Let  $G$  be a group and  $a \in G$ . Prove that the set  $\{a^n \mid n \in \mathbb{Z}\}$  is an abelian subgroup of  $G$ .
7. Draw the multiplication table for the group  $U_{18}$  and find all its generators.
8. Prove that every subgroup of a cyclic group is cyclic.

–Amin Witno