Philadelphia University
Department of Basic Sciences

Exam 1 **Abstract Algebra 1** **15–11–2009**

Choose any 4 problems from the following 6 problems.

1. Let G be a group such that $axb = cxd$ implies $ab = cd$ for all $a, b, c, d, x \in G$. Prove that G is abelian.
2. Let G be the set of all real numbers except -1. Define a binary operation $\star$ on G such that $a \star b = a + b + ab$. Prove that G is a group.
3. Let G be a group and $a \in G$. Prove that the set $\{x \in G \mid xa = ax\}$ is a subgroup of G.
4. Let G be an abelian group with identity e. Prove that the set $\{x \in G \mid x^2 = e\}$ is a subgroup of G.
5. Let G be a group and $H \subseteq G$. Assume that H is a finite set and $ab \in H$ for all $a, b \in H$. Prove that H is a subgroup of G.
6. The group $U_6 \times Z_3$ is cyclic. Find all its generators.

Notes:

1. Full credit will only be given to a solution which is logically correct. Be very careful in what you write!
2. You may assume all the theorems given in the notes, unless when the problem asks you to prove the theorem.
3. Do not spend too much time on a single problem. Read the entire set of problems first; mark the ones you know how to solve and cross out the ones you don't.
4. Do exactly four problems. No bonus points will be given to a fifth solution and beyond. If you have extra time, double check your work.

–Amin Witno