

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2

Abstract Algebra 1

20–12–2009

Choose any 4 problems from the following 6 problems.

1. The group U_{18} is cyclic. Draw its subgroup lattice.
2. Let G be a group and H be a subgroup of G . Let N be another subgroup of G which is normal. Prove that $N \cap H$ is a normal subgroup of H .
3. Let G be a group and H a subgroup of G . Let $a \in G$ and $K = \{aha^{-1} \mid h \in H\}$.
(a) Prove that K is a subgroup of G . (b) Prove that H is isomorphic to K by defining the map $\theta(h) = aha^{-1}$.
4. Draw the Cayley table for the factor group $Z_{20}/\langle 12 \rangle$.
5. Suppose that $\theta : G \rightarrow G'$ is a group homomorphism. (a) What is the meaning of $\ker(\theta)$? (b) Prove that $\ker(\theta)$ is a subgroup of G . (c) Prove that the subgroup $\ker(\theta)$ is normal.
6. Prove that the group U_7 is isomorphic to Z_6 .

Notes:

1. Full credit will only be given to a solution which is logically correct. Be very careful in what you write!
2. You may assume all the theorems given in the notes, unless when the problem asks you to prove the theorem.
3. Do not spend too much time on a single problem. Read the entire set of problems first; mark the ones you know how to solve and cross out the ones you don't.
4. Do exactly four problems. No bonus points will be given to a fifth solution and beyond. If you have extra time, double check your work.