Philadelphia University

Department of Basic Sciences

Final Exam **Abstract Algebra 1** **31–01–2010**

Choose any 5 problems from the following 8 problems.

1. (a) What is the definition of a subgroup? (b) Let G be a group with identity e, and let $a \in G$. Let $H = \{n \in \mathbf{Z} \mid a^n = e\}$. Prove that H is a subgroup of the group $\mathbf{Z}$ of integers.

2. (a) What is the definition of a cyclic group? (b) Prove that a subgroup of a cyclic group is cyclic.

3. Let N be a normal subgroup of a group G. (a) What is the definition of the group G/N? (b) Draw the Cayley table for the group $U_{21}/\langle 4 \rangle$.

4. (a) What is the definition of an isomorphism? (b) Let G be a group, and let $\theta : G \to G$ such that $\theta(a) = a^{-1}$. Prove that θ is an isomorphism if and only if G is abelian.

5. Consider the permutation group $H = \langle\, (1\ 3\ 5\ 7)(2\ 4\ 6)\, \rangle$, which is a cyclic subgroup of the symmetric group S_7. (a) What is the order of H? (b) Draw the subgroup lattice for H.

6. (a) What is the meaning of even permutations? (b) The subset A_n of even permutations is a subgroup of S_n. Prove that A_n is normal.

7. (a) What are the elements of S_2? (b) What are the elements of S_3? (c) What are the elements of A_3? (d) Is $S_3 \approx A_3 \times S_2$? Prove true or false.

8. (a) What is a dihedral group D_n? (b) Let H be a subgroup of D_n. Prove that if the order of H is odd, then H is cyclic.

Note:

1. Full credit will only be given to a solution which is logically correct. Be very careful in what you write!

2. You may assume all the theorems given in the notes, unless when the problem asks you to prove the theorem.

3. Do not spend too much time on a single problem. Read the entire set of problems first; mark the ones you know how to solve and cross out the ones you don't.

4. Do exactly five problems. No bonus points will be given to a sixth solution and beyond. If you have extra time, double check your work.

–Amin Witno