PHILADELPHIA UNIVERSITY

DEPARTMENT OF BASIC SCIENCES

Final Exam **Abstract Algebra 1** **14–01–2013**

Part 1. Each problem is worth 4 points.

1. Is the group Z_{12} cyclic or not cyclic? If cyclic, find all the generators.
2. Draw the Cayley table for the factor group $U_{24}/\langle 7\rangle$.
3. Let $\theta : \mathbb{Z}_{12} \to U_9$ be the homomorphism such that $\theta(n) = 2^n$.
   (a) Find $\theta(5)$.
   (b) Find $\ker(\theta)$.
   (c) Is θ one-to-one? Why or why not?
   (d) Is θ an isomorphism? Why or why not?
4. Let S_6 be the group of permutations with 6 elements. Let $f = (1, 3, 5, 6, 4, 2)$ and $g = (3, 6)(2, 4, 5) \in S_6$.
   (a) Find $f \circ g$.
   (b) Find $g \circ f$.
   (c) Find $g \circ g$.
   (d) Find the order of $g \circ f$ in S_6.

Part 2. Each problem is worth 8 points.

1. Let $S = \{a + b\sqrt{2} \in \mathbb{R}^* \mid a, b \in \mathbb{Q}\}$. Prove that S is a subgroup of $\mathbb{R}^*$.
2. Let $\theta : G \to G'$ be a group homomorphism. Prove that $\ker(\theta)$ is a normal subgroup of G.
3. Let H be the cyclic subgroup of $\mathbb{Z}$ given by $H = \langle 3\rangle$. Let $\theta : \mathbb{Z} \to H$ be defined by $\theta(n) = 3n$. Prove that θ is an isomorphism.

–Amin Witno