PHILADELPHIA UNIVERSITY

DEPARTMENT OF BASIC SCIENCES

Exam 1   **Abstract Algebra 1**   **18–11–2015**

Choose 4 problems from 5. No bonus.
Incomplete solution will not receive full mark.

1. Let $S = \{x \in \mathbb{R} \mid x \neq 1\}$ and define the binary operation $a \star b = ab - a - b + 2$ for all $a, b \in S$. Prove that S is a group.

2. Let A be a group and K be a subgroup of A. Prove: if A is cyclic then K is cyclic.

3. Let T be an abelian group. Let $S = \{a \in T \mid a^3 = e\}$. Prove that S is a subgroup of T.

4. Prove: the group $\mathbb{Z}_4 \times U_3$ is cyclic or not cyclic. If cyclic, find all the generators.

5. Let $F = \{a + b\sqrt{2} \in \mathbb{R}^* \mid a, b \in \mathbb{Q}\}$. Prove that F is a subgroup of $\mathbb{R}^*$.

–Amin Witno