Philadelphia University

Department of Basic Sciences

Final Exam **Abstract Algebra 1** **01–02–2016**

Note: Incomplete solution will not receive full mark.

1. (7 points) Let G be a group and $x \in G$. Let $S = \{a \in G \mid ax = xa\}$. Prove that S is a subgroup of G.

2. (6 points) Find all the cosets in the group D_5 with respect to the subgroup $\langle (1,5)(2,4) \rangle$.

3. (7 points) Let $\theta : G \to G'$ be a group homomorphism.

   (a) Prove that $\ker \theta$ is a subgroup of G.

   (b) Prove that the subgroup $\ker \theta$ is normal.

4. (7 points) Let $G = \{x \in \mathbb{Q} \mid x \neq \frac{1}{2}\}$ and define the binary operation $a \star b = 2ab - a - b + 1$ for all $a, b \in G$.

   (a) Prove that G is a group.

   (b) Prove that the group G is abelian.

5. (6 points) Prove :

   (a) $U_{12} \not\approx U_{10}$

   (b) $U_8 \approx U_{12}$

6. (7 points) Let G be a group and $g \in G$. Let $\theta : G \to G$ such that $\theta(x) = gxg^{-1}$ for all $x \in G$. Prove that θ is an isomorphism.

–Amin Witno