Philadelphia University

Department of Basic Sciences

Final Exam | **Abstract Algebra 1** | **20–06–2022**

1. (6 points) Let G be a group and $k \in G$. Let $H = \{u \in G \mid ku = uk\}$. Prove that H is a subgroup of G.

2. (6 points) Let the group $G = GL(2, \mathbb{R})$ and let the subgroup $H = SL(2, \mathbb{R}) = \{A \in G \mid \det A = \pm 1\}$. Prove that this subgroup H is normal.

3. (7 points) Find all the cosets for the subgroup $H = \langle (1, 4)(2, 3) \rangle$ in the group $G = A_4$.

4. (7 points) Draw the Cayley table for the factor group G/H, where $G = U_{13}$ and $H = \langle 3 \rangle$.

5. (7 points) Let G be a group with identity e and G' a group with identity e'. Let $\theta : G \to G'$ be a homomorphism. Prove that θ is one-to-one if and only if $\ker \theta = \{e\}$.

6. (7 points) Let G be a group and $v \in G$. Let $\theta : G \to G$ such that $\theta(h) = vhv^{-1}$ for all $h \in G$. (a) Prove that θ is a homomorphism. (b) Prove that θ is one-to-one and onto.

–Amin Witno