Philadelphia University

Department of Basic Sciences

Exam 1 **Abstract Algebra 2** **3–4–2007**

There are 6 problems, you choose 4, no more no less.

1. (a) What is the definition of an integral domain?
   (b) What is the definition of a field?
   (c) Prove that a finite integral domain is a field.
   (d) Give an example where (c) is false if the set is infinite.

2. Let $\theta : Z_{15} \to Z_3$ be a ring homomorphism given by $\theta([a]_{15}) = [a]_3$.
   (a) What is the definition of kernel? Find $\ker(\theta)$.
   (b) What is the definition of one-to-one? Is θ one-to-one?
   (c) What is the definition of onto? Is θ onto?
   (d) What is the definition of a factor ring? Find the elements of the factor ring $Z_{15}/\ker(\theta)$. What is this isomorphic to?

3. Let R be a ring. Prove the following statements, in details.
   (a) $0a = 0$ for every $a \in R$.
   (b) $a(-b) = -(ab) = (-a)b$ for every $a, b \in R$.
   (c) If exists, the element $1 \in R$ is unique.

4. (a) What is the definition of an ideal of a ring?
   (b) Prove that if I and J are two ideals of a ring R then the set $I + J = \{i + j \mid i \in I, j \in J\}$ is also an ideal of R.

5. (a) What is the definition of an ideal of a ring?
   (b) Prove that if I is an ideal a ring R then the set $J = \{r \in R \mid ra = 0 \forall a \in I\}$ is also an ideal of R.

6. (a) What is the definition of the unity of a ring?
   (b) What is the definition of a field?
   (c) Prove that if F is a field and S is a subfield of F, then the unity of S is the same as the unity of F.
   (d) Give an example where (c) is false if F is a ring but not a field.

–Amin Witno