Philadelphia University
Department of Basic Sciences

Final Exam **Abstract Algebra 2** **12–6–2007**

There are 8 problems, you choose 5, no more no less.

1. Let R be a commutative ring and $a \in R$. Prove that the set $I = \{r \in R \mid ar = 0\}$ is an ideal of R.
2. Prove that the ring Z_n is a field if and only if n is a prime number.
3. Show that x^3+4x^2+4x+1 is reducible over $\mathrm{Z}_5[x]$ and then factor it into irreducible polynomials.
4. Let F be a field and $f \in F[x]$. Prove that $F[x]/(f)$ is a field if and only if f is an irreducible polynomial.
5. Find a such that $\mathrm{Q}(\sqrt{3}, \sqrt{4}) = \mathrm{Q}(a)$. Prove your answer in detail.
6. Let $a \in K$, an extension field over F, such that $[F(a){:}F] = 7$. Prove that $F(a^3) = F(a)$.
7. Prove that $f = x^2 + 1$ is irreducible over Z_3. Then show that $\mathrm{Z}_3/(f)$ is the field F_9 and construct its multiplication table.
8. The non-zero elements of the field Z_{19} is a cyclic group under multiplication. Find all its generators.

–Amin Witno