Philadelphia University
Department of Basic Sciences

Final Exam **Abstract Algebra 2** **05–06–2008**

There are 9 problems; you choose 6, no more no less.

1. Factor $f(x) = x^3 + 4x^2 + 4x + 1$ into irreducible polynomials in $Z_5[x]$.
2. Find the minimal polynomial of $\sqrt[3]{\sqrt{2} + \sqrt{3}}$ over Q.
3. The polynomial $f(x) = x^2 + 1$ is irreducible in $Z_3[x]$. Use this to construct the multiplication table for F_9, a field with 9 elements.
4. Let R be a commutative ring with unity. Prove that R is a field if and only if R has no ideal except $\{0\}$ and R itself.
5. Let K be a field extension over F and $a \in K$. Prove that a is algebraic over F if and only if $[F(a) : F]$ is finite.
6. Prove that the characteristic of a finite field exists and is a prime number.
7. Let R be a commutative ring and $a \in R$. Prove that the set $I = \{r \in R \mid ar = 0\}$ is an ideal of R.
8. Prove that every complex number is algebraic over the field of real numbers.
9. Let K be an extension field over F. Suppose that $a \in K$ such that $[F(a) : F] = 5$. Prove that $F(a^2) = F(a)$.

–Amin Witno