Philadelphia University

Department of Basic Sciences

Midterm Exam **Abstract Algebra 2** **23–12–2008**

True or false, each problem is worth 2 points.

____1) Every ring has a unity.

____2) Every ring, if $a \neq 0$ and $ab = ac$ then $b = c$.

____3) Every field has no zero divisor.

____4) Every integral domain, if $a \neq 0$ then there is b such that $ab = 1$.

____5) $\mathbf{Z}$ is an integral domain.

____6) $\mathbf{Z}_{13}$ is a field.

____7) If R, S are fields then $R \times S$ is a field.

____8) Every ideal of $\mathbf{Z}$ is $n\mathbf{Z} = \langle n \rangle$ for some integer n.

____9) If $\theta : R \to R'$ is a ring homomorphism then $\ker(\theta)$ is an ideal of R.

____10) If $\theta : R \to R'$ is a ring homomorphism then $\theta(1) = 1$.

Part 2, each problem is worth 5 points.

1. Let $f \in F[x]$ be irreducible.
   (a) What is the meaning that f is irreducible?
   (b) Prove that if $f \mid gh$ then $f \mid g$ or $f \mid h$.
2. Let f and $g \in F[x]$.
   (a) If $\alpha \in F$, prove that $f(x)$ is divisible by $x - \alpha$ if and only if $f(\alpha) = 0$.
   (b) If $F = Z_7$, show that $f(x) = x^3 - 3$ is not divisible by any polynomial of lower degree.
3. Let f and $g \in F[x]$.
   (a) What is the meaning of a greatest common divisor of f and g?
   (b) If $F = Q$, evaluate $\gcd(x^5 + 4x,\ x^3 - x)$.

–Amin Witno