

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam

Abstract Algebra 2

30-05-2012

Part I. Write short answers. Each problem is worth 1 point.

1. Every ring has a unity. True or false?
2. What is the unity of the ring $M(3, \mathbb{R})$?
3. What are the zero divisors in the ring \mathbb{Z}_{30} ?
4. What are the unit elements of the ring $\mathbb{Z}_4 \times \mathbb{Z}_6$?
5. A field cannot have zero divisors. True or false?
6. What is an example of an integral domain that is not a field?
7. The ring $\mathbb{Z}_3 \times \mathbb{Z}_5$ is a field. True or false?
8. What is an example of a principal ideal domain?
9. What are the elements in the factor ring $\mathbb{Z}_{15}/(6)$?
10. The rings \mathbb{Z}_4 and $\mathbb{Z}_2 \times \mathbb{Z}_2$ are isomorphic. True or false?
11. The polynomial ring $\mathbb{Q}[x]$ is a field. True or false?
12. What is the gcd of $x^{27} - 1$ and $x^{15} - 1$ over \mathbb{Q} ?
13. The polynomial $2x^3 - x^2 + 4x - 3$ is irreducible in $\mathbb{Z}_5[x]$. True or false?
14. The polynomial $x^7 - 12$ is irreducible in $\mathbb{Q}[x]$. True or false?
15. Factor the polynomial $x^3 + 2x + 5$ in $\mathbb{Z}_7[x]$, if possible.
16. What is the minimal polynomial of $\sqrt{3} - \sqrt{2} \in \mathbb{R}$ over \mathbb{Q} ?
17. There exists a field of order 6. True or false?
18. Let F be a field of order 16. What is the value of $\chi(F)$?
19. What is an example of a field of order 25?
20. What is $(2x + 1) \times (x + 2)$ in the field $\mathbb{Z}_3[x]/(x^2 + 1)$?

Part II. Write complete proofs. Choose 2 problems from 5.

1. Let $S = \{a + b\sqrt{5} \mid a, b \in \mathbb{Q}\}$. Prove that S is a subfield of \mathbb{R} .
2. Let $\theta : R \rightarrow S$ be a ring homomorphism. Prove that $\ker(\theta)$ is an ideal of R .
3. Let F be a field. Prove that the polynomial ring $F[x]$ is a principal ideal domain.
4. Let F be a field and $f \in F[x]$. Prove that the factor ring $F[x]/(f)$ is a field if and only if f is irreducible.
5. Let F be a finite field with $\chi(F) = 5$, and let $\theta : F \rightarrow F$ such that $\theta(x) = x^5$ for all $x \in F$. Prove that θ is an isomorphism.

–Amin Witno