# Philadelphia University
## Department of Basic Sciences


**Exam 2** **Abstract Algebra 2** **06–05–2014**

Choose five problems. No bonus.

1. Evaluate $\gcd(x^{63}-1, x^{45}-1)$ in $\mathbb{Q}[x]$.
2. Find the minimal polynomial for $a = \sqrt{3} - 2\sqrt{7} \in \mathbb{R}$ over $\mathbb{Q}$.
3. (a) Prove that the polynomial $f = x^2+x+1$ is irreducible in $\mathbb{Z}_2[x]$. (b) Construct the multiplication table for the finite field $\mathbb{Z}_2[x]/(f)$.
4. Let $F$ be a field. Prove that the polynomial ring $F[x]$ is a principal ideal domain.
5. Let $F$ be a field and $f, g, h \in F[x]$ such that $\gcd(f, g) = 1$. Prove that if $f \mid gh$, then $f \mid h$.
6. Let $F$ be a field and $f \in F[x]$. Prove that the factor ring $F[x]/(f)$ is a field if and only if $f$ is irreducible in $F[x]$.
7. Let $F$ be a finite field with $\chi(F) = p$. Prove that there exists $a \in F$ such that $F = \mathbb{Z}_p(a)$.

–Amin Witno