Philadelphia University

Department of Basic Sciences

Final Exam **Abstract Algebra 2** **05–06–2014**

Choose seven problems. No bonus.

1. Let $S = \{a + b\sqrt{7} \mid a, b \in \mathbb{Q}\}$. Prove that S is a subfield of $\mathbb{R}$.
2. Let R be a ring and let $S = \{r \in R \mid ar = ra \text{ for all } a \in R\}$. Prove that S is a subring of R.
3. Let R be any ring (maybe not commutative) and let I be an ideal of R. Let $S = \{r \in R \mid ra = 0 \text{ for all } a \in I\}$. Prove that S is an ideal of R.
4. Let $\theta : \mathbb{Z}_{20} \to \mathbb{Z}_{10}$ such that $\theta(n) = 6n$ for all $n \in \mathbb{Z}_{20}$. Prove that θ is a ring homomorphism and find its kernel. Is θ one-to-one? Is θ onto?
5. Let $f = x^4 + 3x^3 + x^2 + 2x + 1 \in \mathbb{Z}_7[x]$. Factor f completely using irreducible polynomials.
6. Let $f = x^4 + x^2 + 1 \in \mathbb{Z}_2[x]$. Show that the factor ring $\mathbb{Z}_2[x]/(f)$ has a zero divisor.
7. Find an example of a finite field F with 25 elements. Then find an element $a \in F$ that has order 3 in the multiplicative group F^*.
8. Let F be a finite field with $\chi(F) = 3$. Let $\theta : F \to F$ such that $\theta(x) = x^9$ for all $x \in F$. Prove that θ is a ring isomorphism.
9. Let $f = x^4 + x^3 - x + 1$. Prove that f is reducible over $\mathbb{Z}_2$ but irreducible over $\mathbb{Z}_3$. Is f reducible or irreducible over $\mathbb{Q}$?
10. Let F be a finite field with 27 elements. Let $a \in F$ such that $a \notin \{0, 1, 2\}$. Prove that $F = \mathbb{Z}_3(a)$.

–Amin Witno