Philadelphia University

Department of Basic Sciences

Exam 2 **Abstract Algebra 2** **08–05–2016**

1. Let F be a field. Prove that the polynomial ring $F[x]$ is a principal ideal domain.
2. Evaluate $\gcd(x^8 + x^6 + x^2 + 1,\ x^6 + 2x^2 - 2)$ in $\mathbb{Z}_5[x]$.
3. Let $R = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ and $S = \left\{ \begin{pmatrix} a & 2b \\ b & a \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}$. Consider R as a subring of $\mathbb{R}$ and S a subring of $M(2, \mathbb{Z})$. Prove the ring isomorphism $R \approx S$ using the function $\theta(a + b\sqrt{2}) = \begin{pmatrix} a & 2b \\ b & a \end{pmatrix}$.
4. Factor $f = x^3 + x^2 - x + 2$ using irreducible polynomials in $\mathbb{Z}_7[x]$.
5. Prove that $f = 5x^4 - 30x^2 + 60$ is irreducible in $\mathbb{Q}[x]$.

–Amin Witno