Exam 1 **Abstract Algebra 2** **21–11–2016**

Choose 4 problems from the 5 below.
Each complete and correct solution will receive 5 points.

1. Let R be a ring and $x \in R$. Let $S = \{a \in R \mid ax = xa\}$. Prove that S is a subring of R.

2. Let $S = \{a + b\sqrt{5} \mid a, b \in \mathbb{Q}\}$. Prove that S is a subfield of $\mathbb{R}$.

3. Let R be a commutative ring with ideal I. Let $J = \{x \in R \mid xr \in I$ for all $r \in R\}$. Prove that J is an ideal of R.

4. Let $R = \mathbb{Z}_3 \times \mathbb{Z}_4$ with principal ideal $I = ((0,2))$.

   (a) Find the elements of the factor ring R/I.
   (b) Construct the multiplication table for R/I.
   (c) Find all the units and zero divisors in R/I.

5. Let R be a ring with ideal I. Prove that if I contains a unit element, then $I = R$.

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