Philadelphia University
Department of Basic Sciences

Final Exam **Abstract Algebra 2** **25–01–2017**

Choose 5 problems.

1. Prove that if R is a finite integral domain, then R is a field.
2. Let F be a field and $f \in F[x]$. Prove that the factor ring $F[x]/(f)$ is a field if and only if f is irreducible.
3. Prove that $f = x^5 + x^2 + 1$ is irreducible in $\mathbb{Z}_2[x]$.
4. Let $S = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \mid a, b \in \mathbb{R} \right\}$.
   (a) Prove that S is a subring of $M(2, \mathbb{R})$.
   (b) Is S an ideal of $M(2, \mathbb{R})$? Why or why not?
   (c) Prove that every non-zero element in S is a unit.
   (d) Is S a field? Why or why not.
5. Let $a \in \mathbb{R}$ and $I = \{f \in \mathbb{Q}[x] \mid f(a) = 0 \text{ and } f'(a) = 0\}$.
   (a) Prove that I is an ideal in $\mathbb{Q}[x]$.
   (b) Is the ideal I principal? Why or why not?
6. Let F be a finite field with $\chi(F) = p$. Prove that the function $\theta(x) = x^p$ is a ring isomorphism from F to F.
7. Let F be a field with $|F| = 81$. Prove that there exists $a \in F$ such that $\mathbb{Z}_3(a) = F$.

–Amin Witno