Philadelphia University

Department of Basic Sciences

Exam 1 **Abstract Algebra 2** **06–04–2017**

Choose 4 problems.

1. Let $R = M(2, \mathbb{R})$ and $S = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \mid a, b \in \mathbb{R} \right\}$.
   (a) Prove that S is a subring of R.
   (b) Is S an ideal of R? Explain why or why not.
2. Let $S = \{x + y\sqrt{17} \mid x, y \in \mathbb{Q}\}$. Prove that S is a subfield of $\mathbb{R}$.
3. Let R be a commutative ring, and let I be an ideal of R. Let $J = \{x \in R \mid xr \in I$ for all $r \in R\}$. Prove that J is an ideal of R.
4. Let $R = \mathbb{Z}_3 \times \mathbb{Z}_4$ with principal ideal $I = ((0, 2))$.
   (a) Construct the multiplication table for the factor ring R/I.
   (b) Find all the units and zero divisors in R/I.
5. Let $R = \{a+b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ (a subring of $\mathbb{R}$), and let $S = \left\{ \begin{pmatrix} x & 2y \\ y & x \end{pmatrix} \mid x, y \in \mathbb{Z} \right\}$ (a subring of $M(2, \mathbb{Z})$). Let $\theta : R \to S$ be defined by $\theta(a + b\sqrt{2}) = \begin{pmatrix} a & 2b \\ b & a \end{pmatrix}$. Prove that θ is a ring isomorphism.

–Amin Witno