Philadelphia University
Department of Basic Sciences

Final Exam **Abstract Algebra 2** **14–06–2017**

Choose 5 problems and write a complete solution for each one.

1. Let R be any ring (commutative or not commutative) and let $c \in R$. Prove that the set $S = \{r \in R \mid rc = cr\}$ is a subring of R.

2. Let R be any ring (commutative or not commutative) with an ideal I. Prove that the set $S = \{x \in R \mid xa = 0 \text{ for all } a \in I\}$ is an ideal of R.

3. Let F be a field. Prove that the polynomial ring $F[x]$ is a principal ideal domain.

4. Let F be a field and let $f \in F[x]$. Prove that f is irreducible if and only if $F[x]/(f)$ is a field.

5. Let F be a field and let $f, g, h \in F[x]$ such that f is irreducible. Prove that if $f \mid gh$, then either $f \mid g$ or $f \mid h$.

6. Let $f = x^5 + x^4 + 1$ and $g = x^5 + x^3 + 1$. Prove that f is reducible and g is irreducible in $\mathbb{Z}_2[x]$.

7. Let F be a finite field with $\chi(F) = p$, where p is a prime number. Prove that the function $\theta(x) = x^{p^2}$ is a ring isomorphism from F to F.

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