Philadelphia University
Department of Basic Sciences

Exam 2 **Complex Analysis** **13–05–2018**

1. (2 points) Find all the numbers $z \in \mathbb{C}$ such that
$$e^z = -1 + i\sqrt{3}$$
Write your answer in the form $x + iy$, where $x, y \in \mathbb{R}$.

2. (2 points) Evaluate
$$(1 - i)^{2i}$$
using the principal Log. Write your answer in the form $x + iy$, where $x, y \in \mathbb{R}$.

3. (4 points) Prove that
$$\sinh z = \sinh x \cos y + i \cosh x \sin y$$
for all $z = x + iy \in \mathbb{C}$.

4. (4 points) Evaluate $\int_C f(z)\,dz$, where
$$f(z) = f(x + iy) = x^2 - iy^2$$
and C is the curve from $1 + 2i$ to $2 + 8i$ along the parabola $y = 2x^2$. Write your answer in the form $x + iy$, where $x, y \in \mathbb{R}$.

5. (4 points) Use anti-derivative to evaluate
$$\int_C (\sinh z - \cos 2z)\,dz$$
where C is the semi-circle $z(t) = 1 + e^{it}$ for $0 \le t \le \pi$. Write your answer in the form $x + iy$, where $x, y \in \mathbb{R}$.

6. (4 points) Evaluate $\int_C f(z)\,dz$, where
$$f(z) = \begin{cases} \bar{z} & \text{if } \operatorname{Im} z < 0 \\ z^{-1} & \text{if } \operatorname{Im} z > 0 \end{cases}$$
and C is the circle $z(t) = 2e^{it}$ for $0 \le t \le 2\pi$. Write your answer in the form $x + iy$, where $x, y \in \mathbb{R}$.

–Amin Witno