

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Exam 1

Complex Analysis

19–11–2019

Write complete solution.

1. (4 points) Compute  $(-1+i\sqrt{3})^{3/4}$  using principal Log. (Answer in the form  $x+iy$  where  $x, y \in \mathbb{R}$  and simplified.)
2. (4 points) Find all complex numbers  $z$  such that  $z^2 = 13 - 84i$ .
3. (3 points) Compute  $\sin(\pi + i)$  using the definition  $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$ . (Answer in the form  $x + iy$  where  $x, y \in \mathbb{R}$  and simplified.)
4. (3 points) Find real functions  $u(x, y)$  and  $v(x, y)$  such that  $f(z) = z^3 = u + iv$ .
5. (2 points) Prove the limit involving infinity.

$$\lim_{z \rightarrow \infty} \frac{z^3 - 1}{5z^2 + z} = \infty$$

6. (4 points) Prove using the definition of limit (with  $\epsilon$  and  $\delta$ ).

$$\lim_{z \rightarrow 3+2i} 5iz - z = 13i - 13$$

7. Bonus problem (extra 2 points if completely correct, maximum 20 points total):  
Prove that  $\cos^2 z + \sin^2 z = 1$  for all  $z \in \mathbb{C}$  using the definition of  $\cos z$  and  $\sin z$ .

–Amin Witno