

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Exam 2

Complex Analysis

31-12-2019

Each problem is worth 4 points with complete solution.

1. Let  $u(x, y) = y^3 - 3x^2y$ . (a) Prove that  $u$  is harmonic for all  $(x, y)$  and (b) find its harmonic conjugate  $v$  such that  $f = u + iv$  is analytic.
2. Let  $f(z) = e^x(2y^3 + iy^2 - 7i)$ . Use Cauchy Riemann equations to find (a) the domain where  $f'(z)$  exists and (b) find  $f'(z)$ .
3. Let  $f(z) = \text{Log } z$ . Use Cauchy Riemann equations in polar form to prove (a)  $f'(z)$  exists for all  $z \notin (-\infty, 0]$  and (b) prove  $f'(z) = \frac{1}{z}$ .
4. Evaluate  $\int_C f(z)dz$  where  $C$  is the unit circle (positive orientation) and

$$f(z) = \begin{cases} e^{\pi z} & \text{if } \text{Re}(z) \geq 0 \\ 3z^2 & \text{if } \text{Re}(z) < 0 \end{cases}$$

(Final answer must be in the form  $x + iy$ .)

5. Evaluate  $\int_C \bar{z}^2 dz$  where  $C$  is the straight line from 2 to  $2i$ . (Final answer must be in the form  $x + iy$ .)

-Amin Witno