Philadelphia University
Department of Basic Sciences

Final Exam **Complex Analysis** **23–01–2020**

1. Let $u(x, y) = x^4 + y^4 - 6x^2y^2$.
   (a) (2pt) Prove that u is harmonic for all (x, y).
   (b) (4pt) Find $v(x, y)$ such that $f = u + iv$ is entire.
2. (6pt) Let $f(z) = f(x, y) = 2x^2 + ix + 2ixy - y^3 - iy^2$. Use Cauchy-Riemann equations to find the domain where $f'(z)$ exists and find the derivative.
3. (5pt) Evaluate $\int_C \cos z\, dz$ where C is the semi-circle $e^{it}(-\frac{\pi}{2} \leq t \leq \frac{\pi}{2})$. (Final answer must be in the form $x + iy$, where x, y real numbers.)
4. (5pt) Evaluate $\int_C \bar{z}\, dz$ where C is the straight line from 1 to i. (Final answer must be in the form $x + iy$, where x, y real numbers.)
5. (6pt) Evaluate $\int_C \frac{dz}{z^4+iz^3}$ using Cauchy Integral Formula, where C is the circle $|z| = 3$. (Final answer must be in the form $x + iy$, where x, y real numbers.)
6. (6pt) Evaluate $\int_0^{2\pi} \frac{dx}{5+3\cos x}$ using Cauchy Integral Formula.
7. (6pt) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^2}$ using Cauchy Integral Formula.
8. (2pt) Bonus Problem: (Only if the solution is correct and complete. Maximum mark for this Exam is 40 points.) Use Cauchy-Riemann equations to prove that if $f'(z) = 0$ in some complex domain, then $f(z)$ is constant there.

–Amin Witno