

Department of Basic Sciences — Philadelphia University

Final Exam

Complex Analysis

30-01-2022

There are (8) questions, each is worth 5 points.

- (a) Evaluate and write the final answer in rectangular form: $\frac{-2 + 3i}{3 + i}$
(b) Evaluate and write the final answer in rectangular form: $(-1 - i)^{11}$
- (a) Evaluate and write the final answer in rectangular form: $\cos\left(\frac{\pi}{6} + i\right)$
(b) Evaluate and write the final answer in rectangular form: $(-i)^{-1+i}$ (Principal)
- Let $f(z) = f(x + yi) = e^x y^2 + i e^x y - 3i e^x$. Use Cauchy-Riemann equations to find the domain where $f'(z)$ exists, then find $f'(z)$.
- Prove that $u(x, y) = y^3 - 3yx^2 + 2y$ is harmonic for all $x, y \in \mathbb{R}$ and find a harmonic conjugate v (such that $f(z) = u + iv$ is entire).
- Evaluate the contour integral, where C is the straight line from $z = -2$ to $z = i$ and write the final answer in rectangular form.

$$\int_C (3z^2 + \bar{z}) dz$$

- Evaluate using Cauchy Integral Formula, where C is the circle $z(t) = 2i + \frac{3}{2}e^{it}$, ($0 \leq t \leq 2\pi$) and write the final answer in rectangular form.

$$\int_C \frac{z - 1}{(z^2 - iz)^3} dz$$

- Evaluate using Cauchy Integral Formula

$$\int_0^{2\pi} \frac{dx}{5 - 4 \sin x}$$

- Evaluate using Cauchy Integral Formula

$$\int_0^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 4)}$$