

Department of Basic Sciences — Philadelphia University

Midterm Exam

Complex Analysis

22–05–2022

1. (2 points) Evaluate $\frac{5+8i}{-3+i}$ in the rectangular form $(X) + (Y)i$.
2. (2 points) Evaluate $\text{Log}(-\sqrt{3} + i)$ in the rectangular form $(X) + (Y)i$.
3. (2 points) Evaluate the Principal power $(-i)^{3i}$ in the rectangular form $(X) + (Y)i$.
4. (3 points) Evaluate $(-1 + i)^{14}$ —Final answer in rectangular form $(X) + (Y)i$.
5. (3 points) Let $f(z) = e^{1/z}$. Find the real functions $u(x, y)$ and $v(x, y)$ such that $f(z) = u + iv$.
6. (5 points) Find all complex numbers z such that $z^2 = 21 - 20i$.
7. (3 points) Use the definition $\cosh z = \frac{e^z + e^{-z}}{2}$ and $\sinh z = \frac{e^z - e^{-z}}{2}$ to prove

$$\cosh(z_1 + z_2) = \cosh z_1 \cosh z_2 + \sinh z_1 \sinh z_2$$

8. (3 points) Use the formula $\sin z = \sin x \cosh y + i \cos x \sinh y$ to prove

$$|\sin z|^2 = \sin^2 x + \sinh^2 y$$

9. (2 points) Prove the limit at infinity: $\lim_{z \rightarrow \infty} \frac{iz^4 - 2z + 5i}{z^2 - i} = \infty$
10. (5 points) Let $f(z) = e^x(y^2 + iy^2 - 8i)$. Use Cauchy-Riemann equations to find the domain where $f'(z)$ exists, then find $f'(z)$.