Department of Basic Sciences — Philadelphia University

Final Exam **Complex Analysis** **28–06–2022**

Choose 7 questions from 8. No bonus.

1. Prove that $\cos^2 z + \sin^2 z = 1$ using the definition of $\cos z$ and $\sin z$.

2. Prove that $u(x, y) = y^3 - 3x^2y$ is harmonic for all $x, y \in \mathbb{R}$ and find its harmonic conjugate $v(x, y)$, such that $f(z) = u + iv$ is entire.

3. Let $f(z) = f(x+yi) = 2x^2+ix+2ixy-y^3-iy^2$. Use Cauchy-Riemann equations to find the domain where $f'(z)$ exists and find it.

4. Let $f(z) = (\bar{z})^5$. Use Cauchy-Riemann equations in polar form to find the domain where $f'(z)$ exists and find it.

5. Evaluate the line integral using definition or using anti-derivative.

$$\int_0^2 (t+i)(t^2+2it)^2 \, dt$$

6. Evaluate the contour integral, where C is the straight line from $z = -1$ to $z = 1 + 4i$.

$$\int_C (3z + 2\bar{z}) \, dz$$

7. Evaluate using Cauchy Integral Formula, where C is the circle with center at $z_0 = 0$ and radius $R = 3$.

$$\int_C \frac{z+1}{(z^2+4)} \, dz$$

8. Evaluate using the general form of Cauchy Integral Formula, where C is the circle with center at $z_0 = i$ and radius $R = \frac{1}{2}$.

$$\int_C \frac{z-1}{(z^2-iz)^3} \, dz$$

–Amin Witno