



PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam A

DISCRETE STRUCTURES

30-05-2012

PART (I) Each problem is worth 2 points. Circle one answer.

1) Which proposition is equivalent to $\neg p \vee \neg q$?

- a) $q \rightarrow p$ b) $p \rightarrow q$ c) $p \rightarrow \neg q$ d) $\neg q \rightarrow p$

2) $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$ and $C = \{1, 2, 3\}$. Which set is $\{2, 5, 6\}$?

- a) $(A - B) \oplus C$ b) $(C - B) \oplus A$
c) $(A - C) \oplus B$ d) $(B - A) \oplus C$

3) Which matrix represents an equivalence relation?

- a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

4) Let $R = \{(1,4), (2,3), (3,1), (4,2)\}$. Which matrix represents R^2 ?

- a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5) How many non-negative integer solutions to $A + B + C + D = 12$ with the conditions that $A \geq 5$ and $B \geq 3$?

- a) 10 b) 20 c) 35 d) 56

6) How many permutations with A, B, C, D, E, F which do not contain "FED" ?

- a) 96 b) 114 c) 600 d) 696

7) Which graph is an Euler circuit?

- a) K_5 b) $K_{2,5}$ c) $K_{3,4}$ d) K_4

8) Which graph has adjacency matrix $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$?

- a) C4 b) P4 c) K2,2 d) K1,3

9) Convert the incidence matrix $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ to distance matrix.

- a) $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 2 \\ 1 & 3 & 2 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 2 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$

10) Which graph has diameter 3 ?

- a) K5 b) C7 c) K4,3 d) P5

PART (II) Each problem is worth 4 points. Write complete solutions.

11) Convert the proposition $(p \oplus q) \rightarrow r$ to CNF.

12) How many integers from 1 to 1000 are multiples of 8 or 28 ?

13) Evaluate GCD (987, 654).

14) Let $A = \{2, 3, 10, 20, 30\}$ and $R = \{(a,b) \mid b \bmod a = 0\}$. Draw the Hasse diagram.

15) Solve the Chinese Postman Problem (CPP) for the graph below.

