PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam A **DISCRETE STRUCTURES** **14–06–2014**

PART (I) Each problem is worth 2 points. Circle one answer.

1) A = {1, 2, 3, 4, 5} and B = {2, 4, 6} and C = {1, 2, 3}. Which set is {2, 4, 5} ?

a) $(A - B) \oplus C$ b) $(C - B) \oplus A$
c) $(A - C) \oplus B$ d) $(B - A) \oplus C$

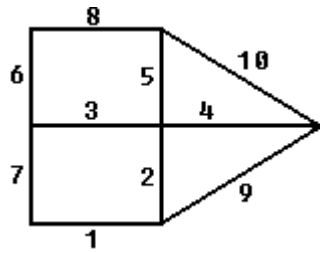
2) Let R = { (1,4), (2,1), (3,2), (4,1) }. Which matrix represents R^2 ?

a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3) Convert the proposition $(p \wedge q) \vee (\neg p \wedge \neg q)$ to CNF.

a) $(p \vee \neg q) \wedge (\neg p \vee \neg q)$ b) $(p \vee q) \wedge (\neg p \vee \neg q)$
c) $(p \vee \neg q) \wedge (\neg p \vee q)$ d) $(p \vee q) \wedge (\neg p \vee q)$

4) What is the value of the minimal spanning tree for this graph?



a) 21 b) 22 c) 23 d) 24

5) How many integers from 1 to 1000 are multiples of 8 and not of 6 ?

a) 63 b) 84 c) 126 d) 167

6) Which graph has the largest degree?

a) P22 b) K5,6 c) K10 d) C11

7) Which graph has an Euler path, not circuit?

a) K_6 b) K_9 c) K_2,9 d) K_1,6

8) Which incidence matrix represents a tree?

a) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

9) Convert the incidence matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ to adjacency matrix.

a) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

10) Which matrix represents a partial order relation?

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

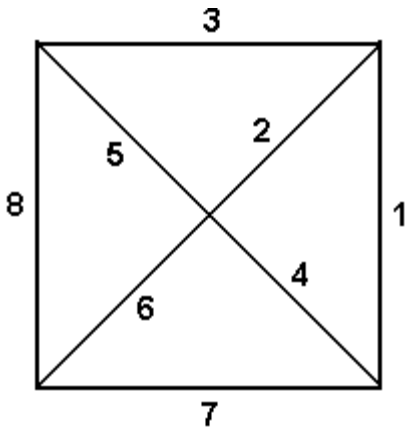
PART (II) Each problem is worth 5 points. Write complete solutions.

11) Evaluate GCD (366, 168) and LCM (366, 168).

12) How many permutations using the letters {A, C, E, M, N, S, T} which do not contain the word CAT or the word MAN?

13) Use induction to prove $2^n < n!$ for all integer $n \geq 4$.

14) Solve the Chinese postman problem (CPP) for the given graph.



--Amin Witno