

Exam 2

Discrete Structures

10–05–2015

Part I. (2 points each) Circle one answer from the multiple choice.

1. The sequence 2, 3, 6, 11, 18, 27, ... is given by the function $S_n =$

- (A) $n^2 + 2$ (B) $4n + 2$ (C) $2^n + 1$ (D) $2n + 2$

2. If $R = \{(1, 3), (2, 1), (3, 1)\}$ then $R^{-2} =$

- (A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. The matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ represents a relation that is

- (A) reflexive (F); symmetric (F); anti-symmetric (F); transitive (T)
 (B) reflexive (F); symmetric (F); anti-symmetric (F); transitive (F)
 (C) reflexive (F); symmetric (F); anti-symmetric (T); transitive (T)
 (D) reflexive (F); symmetric (F); anti-symmetric (T); transitive (F)

4. Let $A = \{1, 2, 3, 4\}$. Which relation on A is a total order?

- (A) $R = \{(a, b) \in A \times A \mid a < b\}$ (B) $R = \{(a, b) \in A \times A \mid a = b\}$
 (C) $R = \{(a, b) \in A \times A \mid a \leq b\}$ (D) $R = \{(a, b) \in A \times A \mid a \neq b\}$

Part II. (4 points each) Write complete solutions.

5. Find the function S_n given the following recurrence.

$$\begin{aligned} S_n &= S_{n-1} + 12S_{n-2} \\ S_0 &= 1 \\ S_1 &= 2 \end{aligned}$$

6. Prove the following formula for all integers $n \geq 1$.

$$1 + 9 + 81 + \dots + 9^n = \frac{9^{n+1} - 1}{8}$$

7. Let $A = \{2, 3, 4, 6, 12\}$ and $R = \{(a, b) \in A \times A \mid a \bmod b = 0\}$.

- (a) Draw the graph for the relation R .
 (b) Prove that R is a partial order relation.
 (c) Draw the Hasse diagram for R .