**D**epartment of Basic Sciences — Philadelphia University

**Final Exam** **Discrete Structures** **24–06–2015**

Part I. (2 points each) Circle one answer from the multiple choice.

1. Convert the proposition $(\neg P \wedge Q) \vee (P \wedge Q)$ to CNF.

   (A) $(\neg P \vee Q) \wedge (P \vee Q)$ (B) $(\neg P \vee Q) \wedge (\neg P \vee \neg Q)$

   (C) $(P \vee \neg Q) \wedge (P \vee Q)$ (D) $(P \vee \neg Q) \wedge (\neg P \vee \neg Q)$

2. Which set equals $(A \oplus B) \oplus A$?

   (A) $A$ (B) $B$ (C) $A - B$ (D) $B - A$

3. From 1 to 200, how many are multiples of 9 or 12?

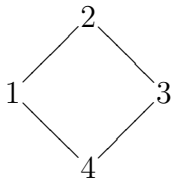
   (A) 31 (B) 33 (C) 40 (D) 41

4. How many permutations we have using the letters A, A, B, B, C?

   (A) 10 (B) 20 (C) 30 (D) 60

5. Which matrix represents the relation $R = \{(a, b) \mid a \bmod 3 = b \bmod 2\}$?

   (A) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

6. Convert the Hasse diagram 

to matrix.

   (A) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

7. Which graph has diameter equals 4?

   (A) $K_5$ (B) $P_4$ (C) $K_{4,4}$ (D) $C_9$

8. Convert the incidence matrix $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ to adjacency matrix.

(A) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

9. Which matrix is the distance matrix of $C_4$?

(A) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 2 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$

10. Which graph is an Euler circuit?

(A) $K_{2,5}$ (B) $K_{3,4}$ (C) $K_6$ (D) $K_7$

Part II. (5 points each) Write complete solutions.

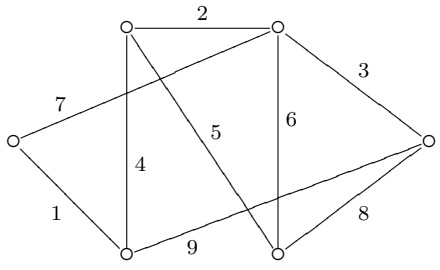
11. Evaluate GCD$(123, 36)$ and LCM$(123, 36)$.

12. Find the function $S_n$ given the recurrence $S_n = 4S_{n-1} - 4S_{n-2}$ with initial conditions $S_0 = 1$ and $S_1 = 8$.

13. Given the matrix for the relation $R$, find the matrix for the transitive closure $\overline{R}$.

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

14. Solve the Chinese Postman Problem (CPP) for the given graph.




–Amin Witno