Department of Basic Sciences — Philadelphia University

Exam 2 **Discrete Structures** **12–05–2016**

Part I. (2 points each) Circle one answer from the multiple choice.

1. The sequence 4, 5, 12, 31, 68, … is given by the function $S_n =$

   (A) $n^2 + 4$ (B) $4n + 4$ (C) $n^3 + 4$ (D) $3n + 4$

2. If $R = \{(1,4),(2,1),(3,2),(4,3)\}$ then $R^3 =$

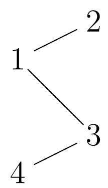
   (A) $\{(1,4),(2,1),(3,2),(4,3)\}$ (B) $\{(1,3),(2,4),(3,2),(4,1)\}$
   (C) $\{(1,2),(2,4),(3,1),(4,3)\}$ (D) $\{(1,2),(2,3),(3,4),(4,1)\}$

3. The matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ represents a relation that is

   (A) reflexive (F); symmetric (F); anti-symmetric (F); transitive (T)
   (B) reflexive (F); symmetric (F); anti-symmetric (F); transitive (F)
   (C) reflexive (F); symmetric (F); anti-symmetric (T); transitive (T)
   (D) reflexive (F); symmetric (F); anti-symmetric (T); transitive (F)

4. Let $A = \{1,2,3,4\}$. Which relation on A is a total order?

   (A) $R = \{(a,b) \mid b > a\}$ (B) $R = \{(a,b) \mid a \bmod b = 1\}$
   (C) $R = \{(a,b) \mid a \geq b\}$ (D) $R = \{(a,b) \mid a \bmod b = 0\}$

5. Convert the Hasse diagram  to matrix.

   (A) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Part II. (10 points total) Write complete solutions.

6. Find the function S_n given the following recurrence $S_n = 2S_{n-1} + 15S_{n-2}$ with $S_0 = 1$ and $S_1 = 2$.

7. Use induction to prove the following formula for all integers $n \geq 1$.

$$1 + 6 + 36 + \cdots + 6^n = \frac{6^{n+1} - 1}{5}$$

8. Let $A = \{2,3,4,5,8,9\}$ and $R = \{(x,y) \mid x \bmod 3 = y \bmod 3\}$. (a) Draw the graph for this equivalence relation. (b) Find the equivalence classes.

–Amin Witno