Department of Basic Sciences — Philadelphia University

Exam 2 **Discrete Structures** **05–01–2017**

Part I. (1 point each) Multiple choice: circle one answer.

1. The sequence 3, 4, 11, 30, 67, ... is given by the function

   (A) $n^2 + 3$ (B) $2^n + 2$ (C) $n^3 + 3$ (D) $3^n - 2^n + 3$

2. The sequence 1, 1, 5, 9, 29, ... is given by the recurrence relation

   (A) $f(n) = 2f(n-1) + 3f(n-2)$ (B) $f(n) = f(n-1) + 4f(n-2)$
   (C) $f(n) = 3f(n-1) + 2f(n-2)$ (D) $f(n) = 4f(n-1) + f(n-2)$

3. The relation $R = \{(x, y) \mid x \bmod y \geq 1\}$ is represented by the matrix

   (A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

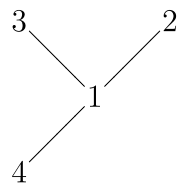
4. The relation $R = \{(1, 1), (1, 3), (2, 2), (3, 1)\}$ is

   (A) symmetric (F); transitive (T) (B) symmetric (F); transitive (F)
   (C) symmetric (T); transitive (T) (D) symmetric (T); transitive (F)

5. For $A = \{1, 3, 5, 8, 10\}$, the equivalence classes associated with the equivalence relation $R = \{(x, y) \mid (x - y) \bmod 2 = 0\}$ are

   (A) $\{1, 3, 5\}, \{8, 10\}$ (B) $\{1, 3\}, \{6, 8, 10\}$
   (C) $\{1, 10\}, \{3\}, \{5, 8\}$ (D) $\{1, 10\}, \{3, 6\}, \{8\}$

6. The given Hasse diagram coincides with the matrix



   (A) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

7. A total order relation is represented by the matrix

   (A) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Part II. (13 points) Complete solution: write the answers in the space provided.

8. Find the function $f(n)$ given by its recurrence relation.

$$\begin{cases} f(n) = 6f(n-1) - 9f(n-2) \\ f(0) = 2 \\ f(1) = 7 \end{cases}$$

9. Use induction to prove the following formula for all integers $n \geq 1$.

$$1 + 2 + 3 + 4 + \cdots + (n+1) = \frac{(n+2)(n+1)}{2}$$

10. Given the relation R, find the matrix for the transitive closure $\overline{R}$.

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

–Amin Witno