Department of Basic Sciences — Philadelphia University

**Exam 2** **Discrete Structures** **27–12–2018**

Part I. (1 point each) Multiple choice; Circle one answer.

1. Let $|A| = 27$. How many subsets of $A$ have 2 elements?

(A) 325 (B) 351 (C) 378 (D) 406

2. Count how many non-negative integer solutions of $A + B + C + D = 30$ with condition $A \geq 13$. Answer in $C(n, k)$:

(A) $C(19, 3)$ (B) $C(20, 3)$ (C) $C(21, 3)$ (D) $C(22, 3)$

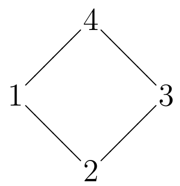
3. Find a function that gives the sequence $3, 4, 7, 12, 19, \ldots$

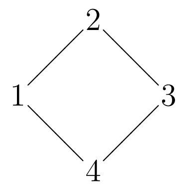
(A) $n^2 + 3$ (B) $n^2 + 4$ (C) $2^n + 2$ (D) $2^n + 3$

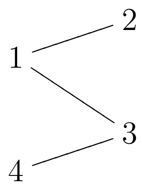
4. Given $R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ compute the matrix $R \circ S$.

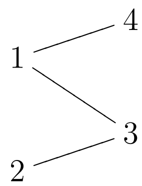
(A) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5. Given the relation matrix $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ draw the Hasse diagram.

(a)



(b)



(c)



(d)



6. Given the incidence matrix $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ find the adjacency matrix.

(A) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Part II. Write your solutions in the space provided.

7. (1 point) Given $S(0) = 0$, $S(1) = 1$ and $S(n) = 2S(n-1) + 3S(n-2)$, find $S(5)$

8. (2 points) Count how many non-negative integer solutions of $A+B+C = 10$ with condition $A \leq 5$.

9. (3 points) Given $R = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ find the matrix for the transitive closure $\overline{R}$.

10. (4 points) Use induction and prove the formula for all integers $n \geq 1$.

$$1 + 7 + 49 + \cdots + 7^{n+1} = \frac{7^{n+2} - 1}{6}$$

11. (4 points) Find the function $S(n)$ given the recurrence relation

$$S(n) = 2S(n-1) + 15S(n-2)$$

with $S(0) = 1$ and $S(1) = 2$.

–Amin Witno