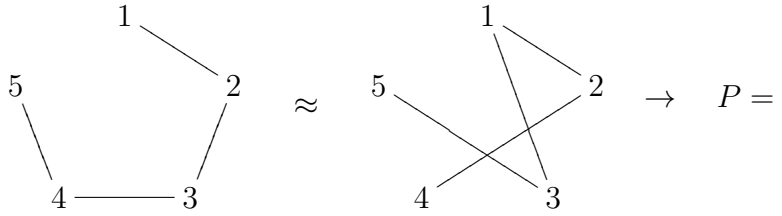
Department of Basic Sciences — Philadelphia University

Midterm Exam **Graph Theory** **17–05–2022**

1. (1 point) G is an 8-regular graph with 44 edges. Find $|V_G|$.
2. (1 point) The graph G has degree sequence $(6, 5, 4, 4, 3, 2, 2, 1, 1)$. Find $|E_G|$.
3. (1 point) G is a regular complete bipartite graph with $|V_G| = 14$. Find $|E_G|$.
4. (1 point) The degree sequence of G is $(9, 8, 8, 6, 5, 5, 4, 3, 3, 1)$. Find $\deg \overline{G}$.
5. (1 point) The graph G is self-complementary with 21 vertices. Find $|E_G|$.
6. (1 points) G is a tree with $\deg G = 30$. Find $|V_G|$.
7. (1 point) Given the incidence matrix Z of a graph, find the adjacency matrix A.

$$Z = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

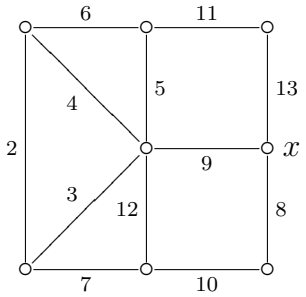
8. (1 point) Two graphs below are isomorphic with adjacency matrices A and B. Find a permutation matrix P (without proof) such that $PAP^T = B$.



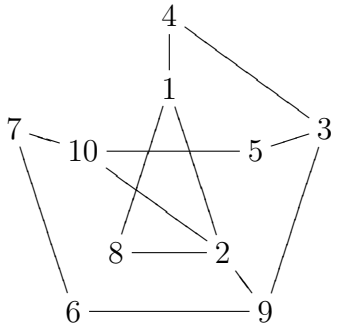
9. (1 point) Find $\deg(\overline{K_{4,5}})$.
10. (3 points) Apply the Degree Sequence Algorithm for the sequence $(8, 5, 4, 4, 3, 2, 2, 1, 1)$
11. (3 points) Given the weight matrix W. Find the total weight of the Minimal Spanning Tree using Kruskal algorithm.

$$W = \begin{bmatrix} 0 & 8 & 0 & 9 & 12 \\ 8 & 0 & 7 & 6 & 0 \\ 0 & 7 & 0 & 5 & 10 \\ 9 & 6 & 5 & 0 & 11 \\ 12 & 0 & 10 & 11 & 0 \end{bmatrix}$$

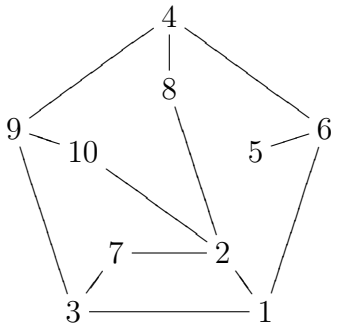
12. (3 points) (a) Draw the Minimal Spanning Tree using Prim algorithm starting at vertex x then (b) Write the MST sequence.



13. (3 points) Apply the BFS algorithm starting at vertex 8. Then (a) Write the BFS sequence and (b) Draw the spanning tree.



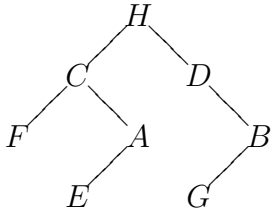
14. (3 points) Apply the DFS algorithm starting at vertex 2. Then (a) Write the DFS sequence and (b) Draw the spanning tree.



15. (3 points) Given the adjacency matrix A. Use cofactor to count the number of spanning trees of the graph.

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

16. (3 points) Given the labeled binary tree. Find the output using (a) Pre-Order (b) Post-Order (c) In-Order algorithm.



–Amin Witno