Midterm Exam

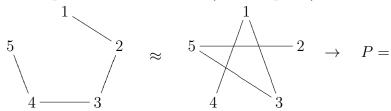
Graph Theory

06/12/2022

- 1. (1 point) A graph is 4-regular and |E| = 36. Find |V|.
- 2. (1 point) Given the degree sequence (6, 5, 4, 4, 3, 3, 2, 2, 1). Find |E|.
- 3. (1 point) G is a tree and deg G = 30. Find $|V_G|$.
- 4. (1 point) The degree sequence of G is (9, 8, 6, 6, 5, 5, 4, 3, 3, 1). Find deg \overline{G} .
- 5. (1 point) A graph is self-complementary and |V| = 20. Find |E|.
- 6. (1 point) Find $deg(\overline{K_{4,6}})$.
- 7. (1 point) Given the incidence matrix Z of a graph, find the adjacency matrix A.

$$Z = \left[\begin{array}{ccccc} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right]$$

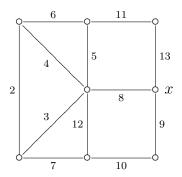
8. (1 point) Two graphs below are isomorphic with adjacency matrices A and B. Find a permutation matrix P (without proof) such that $PBP^T = A$.



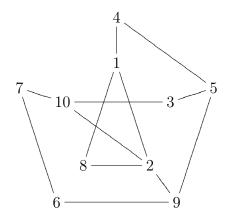
- 9. (3 points) Graphical/not graphical by degree sequence algorithm? (5, 5, 4, 4, 2, 1, 1)
- 10. (3 points) Given the weight matrix W. (a) Draw the graph and (b) Draw the Minimal Spanning Tree using Kruskal algorithm and (c) Write the MST sequence.

$$W = \begin{bmatrix} 0 & 8 & 0 & 9 & 13 \\ 8 & 0 & 7 & 6 & 0 \\ 0 & 7 & 0 & 5 & 11 \\ 9 & 6 & 5 & 0 & 12 \\ 13 & 0 & 11 & 12 & 0 \end{bmatrix}$$

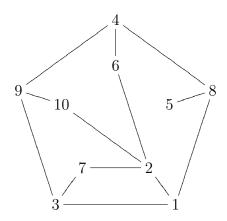
11. (3 points) (a) Draw the Minimal Spanning Tree using Prim algorithm starting at vertex x then (b) Write the MST sequence.



12. (3 points) Apply the BFS algorithm starting at vertex 8. Then (a) Write the BFS sequence and (b) Draw the spanning tree.



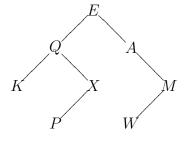
13. (3 points) Apply the DFS algorithm starting at vertex 2. Then (a) Write the DFS sequence and (b) Draw the spanning tree.



14. (4 points) Given the adjacency matrix A. Use cofactor to count the number of spanning trees of the graph.

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

15. (3 points) Find the result from the labeled binary tree.



- (a) Pre-Order:
- (b) Post-Order:
- (c) In-Order:

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