Philadelphia University
Department of Basic Sciences

Linear Algebra II [Exam 1] 6–4–2006

Each problem is worth 4 points.

1. Is W a subspace of V?
   (a) $V = \mathbf{R}^2$ and $W = \{(a - b, ab) \mid a, b \in \mathbf{R}\}$
   (b) $V = M_{2\times 2}(\mathbf{R})$ and $W = \{\begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix} \mid b, c \in \mathbf{R}\}$
2. Does S span V? Is S linearly independent? Is S a basis for V?
   (a) $V = \mathbf{R}^3$ and $S = \{(1, 2, 1), (2, 0, 0)\}$
   (b) $V = \mathbf{R}^2$ and $S = \{(1, 3), (-2, 4), (-2, -6), (3, -6)\}$
3. Is $T : V \to W$ linear?
   (a) $V = W = \mathbf{R}^3$ and $T(x, y, z) = (x - y, 1, y + z)$
   (b) $V = M_{2\times 2}(\mathbf{R}), W = \mathbf{R}$, and $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$
4. Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be a linear transformation given by $T(x, y, z) = (x - y, 2z)$.
   (a) Find $N(T)$ and $R(T)$.
   (b) Is T one-to-one?
   (c) Verify that nullity(T) + rank(T) = dimension (V).
5. Suppose $T : \mathbf{R}^2 \to \mathbf{R}^3$ is a linear transformation such that $T(2, 3) = (1, 0, 2)$ and $T(6, -1) = (3, -2, 4)$.
   (a) Find $T(2, 8)$.
   (b) Find the matrix of T with respect to the bases $\{(2, 3), (6, -1)\}$ for $\mathbf{R}^2$ and $\{(1, 0, 0), (0, -1, 0), (0, 0, 2)\}$ for $\mathbf{R}^3$.

–Amin Witno–