Philadelphia University
Department of Basic Sciences

Exam 2 **Linear Algebra 2** **24–04–2019**

Choose 6 problems out of 7.

1. (3 points) Given two bases $B = \{(3,-1),(2,5)\}$ and $B' = \{(-2,4),(1,-3)\}$ for $\mathbb{R}^2$, find the transition matrix from B to B'.

2. (3 points) Find the standard matrix for the transformation in $\mathbb{R}^2$ performed by reflection about the line $y = x$, followed by rotation of 90^o, and then followed by reflection about the x-axis.

3. (3 points) Find the inverse of the linear operator $T(x,y,z) = (x+y, x-z, x+y+z)$ if exists.

4. (3 points) Find all the eigenvalues of the matrix $A = \begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$.

5. (3 points) The matrix $A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ has eigenvalues $\lambda = 1$ and $\lambda = 5$. For each eigenvalue, find a basis for the eigenspace.

6. (5 points) Find a basis for the (a) row space (b) column space (c) null space of the matrix $A = \begin{bmatrix} 1 & -2 & 2 & 3 & 1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & 0 \\ 0 & 0 & -1 & -2 & 2 \end{bmatrix}$ and (d) find the rank and nullity.

7. (5 points) Prove that a linear transformation $T : V \to W$ is one-to-one if and only if $\ker(T) = \{\mathbf{0}\}$.

–Amin Witno