Philadelphia University
Department of Basic Sciences

Final Exam | **Linear Algebra 2** | **01–06–2019**

1. This problem has 4 parts.

   (a) (2 points) Let $\mathbb{R}^2$ have the weighted Euclidean inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1 + 5u_2v_2$. If $\mathbf{u} = (1, -2)$ and $\mathbf{v} = (3, k)$ are orthogonal vectors, find the value of k.

   (b) (3 points) Let $\langle \mathbf{u}, \mathbf{v} \rangle = 2$ and $\langle \mathbf{v}, \mathbf{w} \rangle = -3$ and $\langle \mathbf{u}, \mathbf{w} \rangle = -1$ and $\|\mathbf{w}\| = 4$. Compute $\langle 2\mathbf{v} - \mathbf{w}, 3\mathbf{u} + 2\mathbf{w} \rangle$.

   (c) (2 points) Compute the cosine of the angle between $A = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$ using the standard inner product on $M_{2,2}$.

   (d) (3 points) Let $p = e^x$ and $q = 3 - e^x \in C[0, 1]$ with the integral inner product $\langle p, q \rangle = \int_0^1 p(x)q(x)dx$. Compute $d(p, q)$.

2. (7 points) Let $p = 3 + 7x + 23x^2 \in P_2$. Find the coordinate $[p]_B$ relative to the basis $B = \{1 + 3x + 7x^2, 2 + 7x + 4x^2, 2 + 6x + 5x^2\}$ for P_2.

3. (7 points) The matrix $A = \begin{bmatrix} 7 & 4 & -6 & -8 \\ -2 & 10 & -3 & 4 \\ -4 & 5 & 0 & -4 \\ 0 & 1 & -1 & 5 \end{bmatrix}$ has eigenvalue $\lambda = 9$. Find a basis for the eigenspace.

4. (7 points) The matrix $P = \begin{bmatrix} -1 & -1 \\ 1 & 5 \end{bmatrix}$ diagonalizes the matrix $A = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix}$. Use the formula $P^{-1}AP = D$ to compute A^7.

5. This problem has 2 parts.

   (a) (6 points) Determine if the matrix $\begin{bmatrix} -1 & 3 \\ 0 & 0 \end{bmatrix}$ is similar or not similar to the matrix $\begin{bmatrix} -1 & -3 \\ 0 & 0 \end{bmatrix}$.

   (b) (3 points) Prove that if the matrix A is similar to the matrix B, then A^2 is similar to B^2.

–Amin Witno