Department of Basic Sciences — Philadelphia University

Final Exam **Linear Algebra 2** **25–01–2022**

1. (3 points) Given the transition matrix $P_{A\to B} = \begin{bmatrix} 3 & 7 \\ -2 & -5 \end{bmatrix}$ and $P_{B\to C} = \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$.
   Find the transition matrix $P_{C\to A}$

2. (4 points) Given $p = x^2$ and $q = x^2 + x - 1 \in P_2$ with the integral inner product $\langle p, q \rangle = \int_0^1 pq\,dx$.
   Part (a) Find $d(p, q)$
   Part (b) Find $proj_p(q)$

3. (5 points) Find (a) the rank (b) basis for Column Space (c) LDE for the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 & 1 & 2 \\ 2 & 2 & 4 & 3 & 2 & 3 \end{bmatrix}$$

4. (3 points) Given $\langle v, w \rangle = 6$ and $\|v\| = 5$ and $\|w\| = 3$, find $\langle 2v + w, v - 2w \rangle$

5. (4 points) Given $w = (12, -4)$ and the basis $B = \{(1, 2), (1, -2)\}$ for R^2
   (a) Prove that B is orthonormal using the inner product $\langle v, w \rangle = \frac{1}{2}v_1w_1 + \frac{1}{8}v_2w_2$
   Part (b) Find $[w]_B$ using inner product

6. (4 points) Find all the eigenvalues (only eigenvalues) of $A = \begin{bmatrix} 2 & 4 & 0 \\ 11 & -5 & 7 \\ 0 & 0 & 8 \end{bmatrix}$

7. (4 points) Given the eigenvalue $k = 2$ for A, find basis for the eigenspace

$$A = \begin{bmatrix} 16 & 0 & 16 & 0 \\ 20 & 8 & 4 & 0 \\ 6 & 0 & 6 & 20 \\ 1 & 0 & 1 & 3 \end{bmatrix}$$

8. (5 points) Compute A^6 using diagonalization, using the given P.

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}; \quad P = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

9. (3 points) Let $S = \{A \in M_{2,2} \mid \det A \geq 0\}$
   Prove S is a subspace or not a subspace of $M_{2,2}$

10. (4 points) Transform the set $\{(1, 0, 2), (2, 0, 0)\}$ to orthonormal set using the inner product $\langle v, w \rangle = 3v_1w_1 + 2v_2w_2 + v_3w_3$

–Amin Witno