**PHILADELPHIA UNIVERSITY**
**FACULTY OF SCIENCE**
**DEPARTMENT OF BASIC SCIENCES**

**FORM A**

**Linear Algebra**
**Final Exam: 15-6-2004**
**Dr. Sabah Ahmad**
**Dr. Amin Witno**

1. (8 points)
Solve the following system of linear equations using Cramer's rule.

$$\begin{aligned} x &+ 2y &+ z &= 6 \\ x &- 4y &- z &= -11 \\ 3x &- 2y &+ z &= 0 \end{aligned}$$

2. (8 points)
Use Gram-Schmidt process to find an orthonormal basis for $R^3$ from the basis $\{(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}), (\frac{-1}{2}, \frac{1}{2}, 0), (\frac{1}{3}, \frac{1}{3}, \frac{2}{3})\}$.

3. (16 points)
Let $T: R^3 \to R^3$ be the linear operator defined by
$T(x, y, z) = (2x + y - z, 2y + z, -3y - 2z)$.

a) (6 points) Is $T$ one-to-one? If so find $T^{-1}(x, y, z)$.
b) (8 points) Find the eigenvalues and eigenvectors of $T$.
c) (2 points) For each eigenvectors find the rank and nullity.

4. (10 points)
Let $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$. Compute $A^{100}$ using diagonalization.

5. (8 points)
Let $B_1 = \{(2, 7), (-1, -2)\}$ and $B_2 = \{(1, 2), (3, 5)\}$ be two bases for $R^2$.

a) (4 points) Find the matrix of transition from $B_1$ to $B_2$.
b) (4 points) Find the matrix of transition from $B_2$ to $B_1$.