PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

First Exam **NUMBER THEORY** **17–11–2005**

Each problem is worth 2 points.

1. Factor the number 1749 into primes.
2. Does the equation $200x + 300y = 120$ have a solution? Why or why not?
3. Are there infinitely many primes in the sequence 2, 7, 12, 17, 22, 27, 32, 37, ... Why or why not?
4. How many are the positive divisors of the number 300,000?
5. The equation $13x + 9y = 1$ has a solution $x = -11$ and $y = 16$. Find two more solutions.
6. Use Euclidean Algorithm to compute gcd(41, 29).
7. Find a solution of the equation $41x + 29y = \gcd(41, 29)$.
8. Prove that if $m \mid a$ and $n \mid a$ and $\gcd(m,n) = 1$ then $mn \mid a$.
9. Give an example where number (8) is not true if $\gcd(m,n) \neq 1$.
10. Prove that if m and n are odd then $2 \mid m^2 + n^2$ but $4 \nmid m^2 + n^2$.
11. Bonus: Prove that if m and n are odd then $m^2 + n^2$ is not a square, meaning that $m^2 + n^2 = x^2$ has no integer solution for x. Hint: use number (10) and uniqueness of prime factorization.

Primes < 100
2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97