

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2

Number Theory

6-5-2007

Each problem is worth 4 points. Solutions must be complete to receive full credit.

1. Write only the final answer for each problem.
 - (a) Evaluate $\phi(120)$.
 - (b) Find the unit digit of 7^{1234} .
 - (c) Find a reduced residue system modulo 18, all prime numbers.
 - (d) A solution of $27x \equiv 18 \pmod{36}$ is $x_0 = 2$. Find the general solution.
2. Compute $70! \% 73$. Note that 73 is prime.
3. I have a little more than 2 dinars left in my mobile phone prepaid account. I could try to spend it all by sending international SMSs, for 6 piasters each, but then 5 piaster would be left. Or I could use it all for MMSs, 13 piasters each, and 12 piasters would be left. How much credits exactly do I have?
4. Solve the congruence $x^5 \equiv 123 \pmod{299}$. Note that $299 = 13 \cdot 23$.
5.
 - (a) Suppose $\gcd(m, n) = 1$. Prove that $a \equiv b \pmod{mn}$ if and only if $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$.
 - (b) Prove that $a^{12} \equiv 1 \pmod{35}$ for any a relatively prime to 35.

–Amin Witno