Philadelphia University
Department of Basic Sciences

Final Exam **Number Theory** **11–6–2007**

Each problem is worth 5 points. Solutions must be complete to receive full credit.

1. Find a, b such that $77a + 176b = \gcd(77, 176)$.
2. Illustrate Fermat Factorization with $n = 7597$.
3. Evaluate $\phi(720)$.
4. Solve the congruence $x^{43} \equiv 17 \pmod{77}$.
5. Suppose $\gcd(a, 77) = 1$. Prove that $a^{30} \equiv 1 \pmod{77}$ with the help of Chinese Remainder Theorem.
6. How many are the primitive roots modulo 17? Show that 11 is one of them.
7. Below is a table for powers of 7 mod 17. Solve the congruence $5^x \equiv 11 \pmod{17}$.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$7^k \% 17$	7	15	3	4	11	9	12	16	10	2	14	13	6	8	5	1

8. Evaluate the Legendre symbol $\left(\frac{a}{p}\right)$ with $a = 47$ and $p = 71$.
9. Solve the quadratic congruence $x^2 \equiv 71 \pmod{77}$.
10. Let g be a primitive root modulo a prime $p > 2$. Prove that g is a quadratic non-residue modulo p.

The list of primes below 200.

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61
67 71 73 79 83 89 97 101 103 107 109 113 127 131 137
139 149 151 157 163 167 173 179 181 191 193 197 199

–Amin Witno