

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2

Number Theory

12-05-2008

Each problem is worth 4 points. Solutions must be complete to receive full credit.

1. Find all integers x satisfying the congruence

$$96x \equiv 56 \pmod{40}$$

2. Compute with the help of Wilson's theorem:

$$100! \% 103$$

Note that 103 is prime.

3. Solve the following system of three congruences:

$$x \equiv 2 \pmod{3}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 4 \pmod{8}$$

4. Prove that if $\gcd(a, 671) = 1$ then

$$a^{60} \equiv 1 \pmod{671}$$

Hint: $671 = 11 \times 61$ and use the Chinese remainder theorem and Fermat's little theorem.

5. Find all the solutions for x such that

$$x^{589} \equiv 2 \pmod{671}$$

Hint: $589^{-1} \equiv 109 \pmod{600}$ and use successive squaring.