Philadelphia University
Department of Basic Sciences

Final Exam **Number Theory** **09–06–2008**

1. Find all the solutions to $405x + 234y = 45$.
2. Solve the following system of three congruences:

$$x \equiv 4 \pmod{5}$$
$$x \equiv 7 \pmod{8}$$
$$x \equiv 8 \pmod{11}$$

3. Find all the solutions to $x^{453} \equiv 2 \pmod{799}$. Note that $799 = 17 \times 47$.
4. Find all the primitive roots modulo 18.
5. Complete the following table and use it to solve $2 \times 7^x \equiv 13 \pmod{17}$.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$5^k \,\%\, 17$																

6. (a) Let $n = 10t + u$. Prove that if $13 \mid (t + 4u)$ then $13 \mid n$.
   (b) Suppose that $\gcd(a, 247) = 1$. Prove that $a^{36} \equiv 1 \pmod{247}$. Note that $247 = 13 \times 19$.

–Amin Witno