

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Final Exam

Number Theory

06–06–2010

Choose six problems only.

1. Find all integer solutions of  $123x + 45y = 66$ .

2. Solve the following system of three congruences:

$$x \equiv 4 \pmod{7}$$

$$x \equiv 3 \pmod{8}$$

$$x \equiv 6 \pmod{9}$$

3. Let  $p$  be a prime number. If  $p \mid mn$ , prove that either  $p \mid m$  or  $p \mid n$ .

4. Find all the primitive roots modulo 22.

5. Evaluate  $2^{8650} \% 10800$  with the help of Euler's theorem.

6. Suppose  $\gcd(a, 161) = 1$ . Prove that  $a^{66} \equiv 1 \pmod{161}$  using the Chinese remainder theorem and Fermat's little theorem. The number  $161 = 7 \times 23$ .

7. Prove there is or there is no solution of  $x^2 \equiv 4x + 318 \pmod{727}$ . The number 727 is prime.

–Amin Witno