Philadelphia University

Department of Basic Sciences

Final Exam **Number Theory** **02–06–2011**

1. Evaluate $\gcd(21560, 4356)$ by (a) the Euclidean algorithm (b) factorization into primes.
2. Given a prime number $p > 2$. Prove that if $p \in [2]_3$ then $p \in [5]_6$.
3. Solve the following system of three congruences.

   $x \equiv 2 \pmod 5$
   $x \equiv 7 \pmod 8$
   $x \equiv 9 \pmod{11}$

4. Prove that $x^{49} \equiv x \pmod{221}$ for all integer x. Note that $221 = 13 \times 17$.
5. Solve the discrete logarithm problem $7^x \equiv 5 \pmod{13}$ using the primitive root $g = 2$.
6. Evaluate the Legendre symbol $\left(\frac{11}{29}\right)$ according to (a) Gauss's lemma (b) the reciprocity law.
7. Solve the quadratic congruence $x^2 \equiv 34 \pmod{55}$. Note that $55 = 5 \times 11$.

–Amin Witno