Philadelphia University

Department of Basic Sciences

Final Exam **Number Theory** **30–05–2012**

Solutions must be complete in order to receive full credit.

1. Find all the integer solutions to $234x + 105y = 27$.
2. Find all integers x, solution to the three congruences.

$$\begin{aligned} x &\equiv 2 \pmod 3 \\ x &\equiv 3 \pmod 4 \\ x &\equiv 1 \pmod 5 \end{aligned}$$

3. Find a reduced residue system (RRS) modulo 18, consisting of only composites.
4. Evaluate $2^{4935} \% 29$ with the help of Euler's theorem. The number 29 is prime.
5. Let n be an integer such that $\gcd(n, 63) = 1$. Use Chinese remainder theorem (CRT) to prove that $n^6 \equiv 1 \pmod{63}$.
6. Count how many primitive roots we have modulo $n = 3125$.
7. Find all the integer solutions to the discrete logarithm problem $5^x \equiv 6 \pmod 7$ using the primitive root $g = 3$.
8. Evaluate the Legendre symbol $\left(\frac{194}{239}\right)$. The number 239 is prime.
9. Find all the integer solutions to the quadratic congruence $x^2 \equiv -9 \pmod{65}$. The number 65 is composite.
10. Let $p > 2$ be a prime number. Prove that if g is a primitive root modulo p, then g is a quadratic nonresidue modulo p.

–Amin Witno