Philadelphia University

Department of Basic Sciences

Final Exam **Number Theory** **15–01–2013**

Part 1. Short Answer, 2 points each.

1. Is the number 493 prime or composite?
2. Find a reduced residue system (RRS) modulo 20 with only prime numbers.
3. Find the inverse of 17 modulo 32.
4. Evaluate $3^{100} \% 17$ using Fermat's little theorem (FLT).
5. Evaluate $\phi(12000)$.
6. Evaluate $|2|_{15}$.
7. Find one primitive root modulo 7.
8. How many primitive roots are there modulo 27?
9. Find all the quadratic non-residues (NR) modulo 11.
10. Evaluate the Legendre symbol $\left(\frac{24}{29}\right)$.

Part 2. Complete Solution, 5 points each.

1. Find all the integer solutions to $2013x + 3102y = 9999$.
2. Let n be an integer such that $\gcd(n, 65) = 1$. Use Chinese remainder theorem (CRT) to prove that $n^{12} \equiv 1 \pmod{65}$.
3. Find all the integer solutions to the discrete logarithm problem $5^x \equiv 4 \pmod{11}$ using the primitive root $g = 7$.
4. Use Chinese remainder theorem (CRT) to find all the integer solutions to the quadratic congruence $x^2 \equiv 23 \pmod{77}$.

–Amin Witno

The list of prime numbers $p < 200$:

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199				